A construction of class fields via periods of toric $K3$ hypersurfaces

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It is an interesting problem in number theory to obtain an explicit construction of class fields (cf. Hilbert's 12th problem). In this talk, we shall see a construction via periods of toric $K3$ hypersurfaces. If a reflexive polytope $P \subset \mathbb{R}^3$ with terminal singularities is given, we can construct a toric 3-fold X_P. We can see that anti-canonical sections of X_P give $K3$ surfaces. We call them *toric $K3$ hypersurfaces*. They are very important in the study of mirror symmetry of $K3$ surfaces.

In this talk, we study a particular toric $K3$ hypersurfaces explicitly given by

$$S_0(\lambda,\mu) : xyz^2(x+y+z+1)+\lambda xyz+\mu=0.$$

Period mappings are important in the study of the moduli of $K3$ surfaces (cf. Torelli's Theorem). In our case, the period mapping Φ for our family $\{S_0(\lambda,\mu)\}$ is given by the multivalued mapping from (λ,μ)-space to the product $\mathbb{H}\times\mathbb{H}$ of the upper half planes. The inverse correspondence

$$\Phi^{-1} : (z_1,z_2) \mapsto (\lambda(z_1,z_2),\mu(z_1,z_2))$$

of the period mapping Φ gives a pair of *Hilbert modular functions* for $\mathbb{Q}(\sqrt{5})$ on $\mathbb{H}\times\mathbb{H}$.

Our construction of class fields gives a non-trivial arithmetic property of the moduli of our toric $K3$ surfaces. Let K be an imaginary quadratic extension of $\mathbb{Q}(\sqrt{5})$ and K^* be its reflex. Let $\mathfrak{O}_K$ be the ring of integers of K. For an F-linear embedding $f : \mathfrak{O}_K \hookrightarrow M_2(F)$, there exists the unique fixed point $(z_1^0,z_2^0) \in \mathbb{H}\times\mathbb{H}$ of $f(K)$.

Theorem 1.*([N])— $K^*(\lambda(z_1^0,z_2^0),\mu(z_1^0,z_2^0))/K^*$ gives an unramified class field.*

Moreover, we have the following precise result.

Theorem 2.*([N])— Let the notation and assumption be as in Theorem 1. Suppose the ideal class group of K is isomorphic to $(\mathbb{Z}/2\mathbb{Z})^{r_1}\oplus(\mathbb{Z}/2^2\mathbb{Z})^{r_2}\oplus\cdots\oplus(\mathbb{Z}/2^k\mathbb{Z})^{r_k}\oplus G_1$, where G_1 does not contain any elements of even order. If $K/\mathbb{Q}$ is a cyclic extension, then $K^*=K$ holds and*

$$\mathrm{Gal}(K(\lambda(z_1^0,z_2^0),\mu(z_1^0,z_2^0))/K) \simeq (\mathbb{Z}/2\mathbb{Z})^{r_2}\oplus\cdots\oplus(\mathbb{Z}/2^{k-1}\mathbb{Z})^{r_k}\oplus G_1.$$

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